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ABSTRACT

The practice of ascribing to agents expectations compat- ible with the model currently proposed by the analyst has been a widespread feature in Macroeconomics. However, that is a problematic assumption when used to depict anticipations constructed in the past since it would imply attributing to agents the use of a model that the economist had not yet built, and possibly not yet thought about. Thus, model-consistency is an ambiguous notion. In this paper we present a preliminary exploration of the application of the alternative forms of model-consistency in a very standard setup, using two re-lated analytical constructs of different generations for U.S. data for the period 1959-2015.

SOBRE EXPECTATIVAS MODELO-CONSISTENTES EN MACROECONOMÍA

RESUMEN

La práctica de considerar que las expectativas de los agentes son compatibles con el modelo actual propuesto por los analistas es común en Macroeconomía. Sin embargo, eso es un supuesto problemático cuando se aplica para las expectativas construidas en el pasado porque implicaría que los agentes usan un modelo que no ha sido desarrollado aun, y probablemente ni siquiera pensado. Entonces, la consistencia de los modelos es un concepto ambiguo. En este trabajo presentamos una exploración preliminar de la aplicación de formas alternativas de consistencia de los modelos en un marco estándar, usando dos construcciones analíticas de diferentes generaciones para los Estados Unidos en el período 1959-2015.

Keywords: Model-consistency - Rational expectations - Forecasts - Non-linear testing - Wald tests
Palabras claves: Consistencia de los modelos - Expectativas racionales - Predicción - Contrastes no lineales Contrastes de Wald

JEL Codes: C12, C52, E47

1 Introduction

The presumption that the expectations of economic agents should be described as derived from the model proposed by the analyst has been a centerpiece of macroeconomic analysis for several decades already. However, model-consistency, and rational expectations itself, are ambiguous, ill-defined concepts, the practical implementation of which raises issues of logical coherence (see, e.g., Heymann and Pascuini, 2017).

In his seminal paper, Muth (1961) formulates the rational expectations hypothesis as: "the expectations of firms (or more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the 'objective' probability distribution of outcomes)." Sargent (2008) states that: "a rational expectations equilibrium is a fixed point of this mapping (from perceived law of motion to actual law of motion)... From a practical perspective, an important property of a rational expectations model is that it imposes a communism of models and expectations. If we define a model as a probability distribution over a sequence of outcomes..., a rational expectations equilibrium asserts that the same model is shared by (1) all of the agents within the model, (2) the econometrician estimating the model, and (3) nature, also known as the data generating mechanism."

But theories have changed over time, indicating an implicit recognition that they have not converged to a definitive representation of the phenomena of interest. Therefore, the laws of motion described by (2) and (3) do not coincide. When comparing (1) and (2), the evolving nature of the theories makes the concept problematic since, in particular, the formulation does not specify whether model- consistency (M-C) should be interpreted in the usual way (M-Ca) as a correspondence of the expectations mechanism used by agents with the current model of the analyst, or else (M-Cb) a correspondence with the model that the analyst (or the theoretical dynasty to which the present modeler belongs) considered the appropriate tool to understand and forecast the matters of interest when the expectations were being formed.[1] In the first version, if there is a presumption of progress in modeling, agents are assumed to have possessed in the past superior knowledge relative to the contemporaneous economist. The second version would correspond to the intuitive argument that the economically relevant beliefs and behaviors at a certain moment are influenced by the professional views current at the time, and would depict agents as learning (and erring) in parallel with the "representative economist" of the current analyst's lineage. Model-consistency would then allow for non-random mistakes in expectations (from the point of view of the now-incumbent model), while implementing the correspon-

[1] This is a special instance of the more general case where the expectations of the agents at a certain moment can be represented as if they are (or were) based on a different model than the one currently proposed by the analyst. In what follows we shall be satisfied with this more lax concept and will not try to identify precisely what would have been contemplate strict "contemporaneous -model- consistent expectations" at different moments.

dence between the contemporaneous views of agents and the "predictions of the- in vogue at the time- economic theory".

Stemming from works like Sargent (1993, 2001) and Evans and Honkapohja (2001), there is a sizable macroeconomic literature which treats agents as analysts who base their expectations on models where the parameters are updated using statistical techniques as new information arrives. However, as far as we know, the notion of M-Cb, which makes the specification of expectations-relevant models evolve with the changes in accepted macro analysis has not been investigated, or even recognized as a more consistent form of the often invoked M-C concept.

In this paper we explore the application of the alternative definitions of model-consistency in a very standard setup, using two related analytical constructs of different generations, both members of a family with wide circulation in their peak times: an older vintage (starting in the late 1970's) monetary model (M1) which stresses the response of output to unanticipated changes in the money supply (à la Bohara, 1991, related to Barro, 1978, Mishkin, 1982a), and a more recent (mid 2000's) small New Keynesian model (M2) where the policy instrument is the interest rate and monopolistic competitors are subject to Calvo-type (1983) restrictions on price movements (see Cho and Moreno, 2006). In both models, agents must form anticipations to determine their behavior; in each case, the original M1 or M2 models represented expectations as in M-Ca, assuming that that those expectations had been based in the past on the current model, and would remain to be based on it in the future.

We develop our applications with information for the U.S. for the period 1959-2015, since the reference models were developed originally for that economy. We assume that M2 was built on the presumption that it possessed (albeit provisionally) a sustained, non- time-contingent validity (in fact, its builders estimated the model for the whole period, 1980- 2015). Taking M1 as the "previous generation" model, we then ask whether, for different time frames, the data rejects, or not, the use of M1 or M2 as expectations- generating instruments while maintaining M2 as the current structural representation of the variables of interest. Our data set covers the period 1959- 2015, and all the regressions are estimated for 20- years rolling windows within that interval.

We consider two structural- model/ expectation- scheme combinations. The first, M-Ca(M2,M2), uses a structure corresponding to the later-vintage model (M2) and takes M-C as meaning that expectations are generated with that same model. The alternative, M-Cb(M2,M1) keeps M2 as the assumed structure, but represents expectations as if they were generated during the relevant time frame through an M-Ca(M1, M1) procedure, that is, respecting the theory embedded in the older model, including its model- consistency assumption, with the corresponding restrictions on parameters. The evaluation of model-consistency of expectations uses non-linear Wald tests derived from estimations with the generalized method of moments (GMM). For the M-Ca(M2, M2) case, the procedure can be summarized as follows. For each time window, the first step is to estimate a data-driven forecasting scheme through a vector autoregressive (VAR) model, using the variables that appear in the M2 model (aggregate output, inflation and the policy interest rate). The predictions generated through the corresponding reduced form equations are fed into the structural M2 model, and the structural parameters are then estimated (here, using an instrumental variables approach to deal with endogeneity issues). The equations of the corresponding reduced form can then be obtained. As for the exercise that applies the M-Cb(M2,M1) notion, the first stage is as before, because it is carried out without reference to theories and, since M2 is maintained as the assumed structure, the second leg (estimating the model's structural parameters with expectations drawn from an unrestricted VAR) is also the same. But, next, model M1 is used to produce expectations in the manner of M-Ca(M1,M1), that is, with the constraints imposed by the specification of the model. Then the corresponding forecasts are used to estimate the structural parameters of M2. In both cases, the final operation is to check whether the parameters of the structural- based set of reduced form equations are significantly different from those of the unrestricted equations found in the first step. If the answer is positive, the hypotheses (either M-Ca(M2,M2) and/or M-Cb(M2,M1)) are considered to be rejected.

Model-consistency tests (of the M-Ca) type were performed some decades ago on M1 models, in different ways. Mishkin (1982a,1982b), for instance, did not estimate a structural form; rather, he focused on checking whether some parameters of the reduced form were zero, as implied by the theory. Concerning this kind of Wald tests, Gregory and Veall (1987) pointed out that they are quite sensitive in small samples to the way in which the non-linear restrictions are parametrized. Alternative tests were proposed by Hoffman and Smith (1981) and Godfrey and Veall (1985a,1985b). Hatcher and Minford (2016) present an alternative strategy for testing M-Ca(M2,M2): the "indirect inference procedures" (see also Smith, 1993, Gourieroux and Mofort, 1996). The suggestion here is to compare the VAR parameters drawn from the data and the mean VAR coefficients estimated from bootstrapped samples from the full macro model, after imposing the restrictions on parameters implied by the theory. In our case, the unconstrained model M2 can be estimated without difficulty, and then Wald tests are then preferred to alternatives such as Lagrange multipliers and likelihood ratio tests. In this regard, it is known that non-linear restrictions on structural parameters may imply complicated (or non-convergent) iteration processes; thus the convenience of using the unrestricted model, as allowed by the approach we follow here. For a broader discussion, see, for example, Hatcher and Minford (2016), Le et al. (2011), Liu and Minford (2014).

In our applications, for most of the time interval under consideration, neither M-Ca(M2,M2) nor M-Cb(M2,M1) is rejected. The tests seem to lack power to discriminate between roughly similar models (and possibly between them and other alternatives) as forecast-generators, in a period of relative tranquility in the economy in question. However, when we consider more recent sample periods, with information after the eruption of the crisis of the late 2000's, the parameters of the estimated model (particularly the output gap equation) become unstable. This behavior seems to invite further analysis. In any case, it should be stressed that our interest here is not to validate or negate particular models, or a specific theory of expectations, but simply to perform a preliminary exercise using the two notions of model-consistency.

This paper is organized as follows. Section 2 describes the macroeconomic models, M1 and M2, on which the exercises will be basedM2 and M1 models. Section 3 develops Wald tests for the restrictions associated with the different versions of model- consistency Section 4 presents the empirical results. Section 5 concludes.

2 The basic M2 and M1 models

2.1 A small NKM model, M2

The model (M2) proposed by Cho and Moreno (2004,2006) consists of a small (three- equation) system with three endogenous variables, π_t, y_t and r_t, standing for inflation, the output gap, and the policy nominal interest rate, respectively. Each equation exhibits persistence effects and has forward-looking terms. As usual with small NKM models, the equations represent the aggregate supply schedule (AS), the demand for goods function (IS), and the rule followed by the monetary authorities to determine the interest rate (MP).

The AS curve is a modification Fuhrer and Moore (1995), and writes the inflation rate as determined by inflationary expectations, an inertial effect of past price increases and the current and lagged output gaps. Thus:

$$\pi_t = \alpha_{AS} + \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda(y_t + y_{t-1}) + \epsilon_{AS_t},$$

where $\epsilon_{AS_t} \sim i.i.d.(0, \sigma^2_{AS})$ is the aggregate supply structural shock. E_t is the expected value operator, conditional on available information at time t.

The IS equation is a typical goods- demand function with habit- persistence effects as in Fuhrer (2000), where the output gap results from the expected future level of aggregate production, lagged output and the ex-ante real interest rate:

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(r_t + E_t \pi_{t+1}) + \epsilon_{IS_t},$$

where $\epsilon_{IS_t} \sim i.i.d.(0, \sigma^2_{IS})$ is the aggregate supply structural shock.

Finally, the monetary policy equation, MP, models the policy nominal interest rate through a reaction function conditional on expected inflation and the output gap, with a smoothing autoregressive term (see Clarida, Galí and Gertler (2000):

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1-\rho)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{MP_t},$$

where $\epsilon_{MP_t} \sim i.i.d.(0, \sigma^2_{MP})$ is the monetary policy structural shock.

The equations can be summarized as

$$\begin{bmatrix} 1 & -\lambda & 0 \\ 0 & 1 & \phi \\ 0 & -(1-\rho)\gamma & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \alpha_{AS} \\ \alpha_{IS} \\ \alpha_{MP} \end{bmatrix} + \begin{bmatrix} \delta & 0 & 0 \\ \phi & \mu & 0 \\ (1-\rho)\beta & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ r_{t+1} \end{bmatrix} + \begin{bmatrix} 1-\delta & \lambda & 0 \\ 0 & 1-\mu & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{AS_t} \\ \epsilon_{IS_t} \\ \epsilon_{MP_t} \end{bmatrix}.$$

In compact matrix notation the model can be expressed as:

$$B_{11} X_t = \alpha + A_{11} E_t X_{t+1} + B_{12} X_{t-1} + \epsilon_t, \tag{1}$$

where $X_t = (\pi_t, y_t, r_t)'$ is the vector of endogenous variables, $B_{11} = \begin{bmatrix} 1 & -\lambda & 0 \\ 0 & 1 & \phi \\ 0 & -(1-\rho)\gamma & 1 \end{bmatrix}$, $A_{11} = \begin{bmatrix} \delta & 0 & 0 \\ \phi & \mu & 0 \\ (1-\rho)\beta & 0 & 0 \end{bmatrix}$, and $B_{12} = \begin{bmatrix} 1-\delta & \lambda & 0 \\ 0 & 1-\mu & 0 \\ 0 & 0 & \rho \end{bmatrix}$ are the coefficients matrices of structural parameters, α is the vector of constants, and $\epsilon_t \sim (0_3, D)$ is the vector of structural errors with D a diagonal variance matrix. Define $\eta = (\delta, \lambda, \mu, \phi, \rho, \beta, \gamma)'$ as the multivalued parameter of interest.

Note that one of the key characteristics of the model is that agents base their decisions on expectations about the future value of the vector X_t, $E_t X_{t+1}$.

2.2 M2-NKM expectations

Cho and Moreno (2006, p.1464, eq. (4)) shows that the assumption of no asymmetric information between the economic agents and the monetary policy authority implies a forecasting mechanism of the form

$$X_{t+1} = const + \Omega X_t + \Gamma \epsilon_{t+1}, \tag{2}$$

where b is 3×1 vector and Ω and Γ are 3×3 matrices.

This can be written as

$$X_{t+1} = const + \Omega X_t + e_{t+1}, \tag{3}$$

where $e_{t+1} = \Gamma \epsilon_{t+1}$. Define $\omega = vec(\Omega) = (\omega_{\pi\pi}, \omega_{\pi y}, \omega_{\pi r}, \omega_{y\pi}, \omega_{yy}, \omega_{yr}, \omega_{r\pi}, \omega_{ry}, \omega_{rr})$ as the autoregressive parameters in (3).

2.3 M1 model and expectations

In the 1970's and early 1980's, a highly prominent family of models, originated from works like Lucas (1972), Sargent and Wallace (1976), Barro (1977), represented the determination of the aggregate supply of goods as a function of monetary (or price) surprises, while macro policy was assumed to operate through changes in the magnitude of the money supply; in addition, the models applied the standard M-Ca presumption for expectations. Given their influence in academic environments and in public discourse during a substantial lapse of time, those theories stand up as candidates for having been used for expectations- formation during at least part of the period of interest.

In order to define a concrete and simple instance (M1) of such models, we use a streamlined version based on Barro (1978), Mishkin (1982a, 1982b) and Bohara (1991). A formal property of the model is that it can be solved recursively for a structural vector autoregression identified through a Cholesky decomposition, with a hierarchy of effects: first the, monetary aggregate is forecasted, allowing to determine aggregate output, next inflation and, finally, interest rates.

The specification of the model starts with a reaction function of the central bank which, as in Bohara (op. cit.), gives the growth rate of the money supply at time t given the values at $t-1$ of cyclical output and the interest rate, and its own lagged value. It can be noted that the equation does not allow for a direct response of monetary policy to the inflation rate. In this, we follow the assumptions made in the previously cited papers, which we take as embodying accepted views at their times: we are not interested in building a model per se, but in identifying what could be seen as a "'representative"' member of the models of the earlier generation. Then:

$$m_t = c_m + \psi_{1my} y_{t-1} + \psi_{1mr} r_{t-1} + \psi_{1mm} m_{t-1} + u_{mt}.$$

Now, the aggregate supply equation expresses the deviation from trend of total output as a function of unanticipated money supply, anticipated money (to allow for possible non- neutralities) and an autoregressive term:

$$y_t = \psi_{y0} + \psi_{1yy} y_{t-1} + \psi_{ym1} E_{t-1} m_t + \psi_{ym2}(m_t - E_{t-1} m_t) + u_{yt},$$

where ψ_{ym1} reflects the non-neutrality of money (as highlighted in Mishkin, op. cit., and subsequent works) and ψ_{ym2} the effect of unexpected shocks on output.

Using the two previous equations, the model-based expectation of the cyclical component of output can be obtained as:

$$\begin{aligned} E_{t-1} y_t &= \psi_{y0} + \psi_{yy} y_{t-1} + \psi_{ym1} E_{t-1} m_t \\ &= c_y + \psi_{ym1}\psi_{m\pi}\pi_{t-1} + (\psi_{yy} + \psi_{ym1}\psi_{my}) y_{t-1} + \psi_{ym1}\psi_{mr} r_{t-1} + \psi_{ym1}\psi_{mm} m_{t-1}. \end{aligned}$$

The inflation rate is derived, as was common at the time, from a simple money demand function in the spirit of the quantity theory. The equation we postulate is:

$$\begin{aligned} \pi_t &= \psi_{\pi 0} + \psi_{\pi y} y_t + \psi_{\pi m} m_t \\ &\quad + \psi_{1\pi\pi}\pi_{t-1} + \psi_{1\pi y} y_{t-1} + \psi_{1\pi m} m_{t-1} + \psi_{1\pi r} r_{t-1} + u_{\pi t}, \end{aligned}$$

and the corresponding inflation expectations determined from the model are:

$$\begin{aligned} E_{t-1}\pi_t &= \psi_{\pi 0} + \psi_{\pi y} E_{t-1} y_t + \psi_{\pi m} E_{t-1} m_t \\ &\quad + \psi_{1\pi\pi}\pi_{t-1} + \psi_{1\pi y} y_{t-1} + \psi_{1\pi m} m_{t-1} + \psi_{1\pi r} r_{t-1}. \end{aligned}$$

The system up to this point allows to define anticipations for the money supply, cyclical output and prices. There is here (as in the references quoted above) no equation determining the interest rate. But that variable is needed when combining the later- vintage M2 model with M1 expectations. For that purpose, we write an unrestricted equation where the interest rate is expressed as depending on the other three variables (and their lags):

$$\begin{aligned} r_t &= \psi_{r0} + \psi_{ry} y_t + \psi_{rm} m_t + \psi_{r\pi} \pi_t \\ &\quad + \psi_{1r\pi} \pi_{t-1} + \psi_{1ry} y_{t-1} + \psi_{1rm} m_{t-1} + \psi_{1rr} r_{t-1} + u_{rt}, \end{aligned}$$

Then, the interest rate forecasts implied by the model would be:

$$\begin{aligned} E_{t-1} r_t &= \psi_{r0} + \psi_{ry} E_{t-1} y_t + \psi_{rm} E_{t-1} m_t + \psi_{r\pi} E_{t-1} \pi_t \\ &\quad + \psi_{1r\pi} \pi_{t-1} + \psi_{1ry} y_{t-1} + \psi_{1rm} m_{t-1} + \psi_{1rr} r_{t-1}. \end{aligned}$$

It follows that the expectations mechanism implementing version model-consistency of the form M-Ca(M1,M1) can be expressed as a function of the ψ parameters above

$$X_{t+1} = const + \Omega(\psi) X_t + e_{t+1}, \tag{4}$$

where Ω depends on the ψ parameters and X_t is the vector of the three endogenous variables used for M2, (π_t, y_t, r_t).

2.4 Model-consistency as parameter restrictions

The model-consistency of the M-Ca(M2,M2) and M-Cb(M2,M1) types implies a simultaneous solution of eqs. (1) and either (3) or (4), respectively. Cho and Moreno (2004,2006) solves a macroeconomic model with M-Ca(M2,M2) by imposing the simultaneous solution of the structural model (perceived law of motion) and the VAR(1) model (understood as the actual law of motion), which implies the restrictions:

$$\Omega = (B_{11} - A_{11}\Omega)^{-1} B_{12}. \tag{5}$$

This condition can be written as a quadratic matrix equation:

$$A_{11}\Omega^2 - B_{11}\Omega + B_{12} = 0_{3\times 3}. \tag{6}$$

Note that this implies nine nonlinear restrictions involving both η and ω, which can be summarized by:

$$a(\theta) = vec(A_{11}\Omega^2 - B_{11}\Omega + B_{12}) = 0_{9\times 1}, \tag{7}$$

where $\theta = (\eta', \omega')'$, η are the structural parameters and ω the VAR(1) reduced-form parameters, as defined above.

Our model-consistency tests ask about the validity of this restriction using Wald specification tests of (7).

3 Wald statistics for model-consistency

The classical framework allows for three different types of specification tests. First, we could implement a likelihood ratio statistic that contrasts the values of the objective function corresponding to the unrestricted model with the model incorporating the restrictions coming from (7). A second option would be to use a Lagrange multiplier statistic based on the score functions derived from the objective functions pertaining only to the restricted model. Or, one could perform a Wald statistic computing the asymptotic distribution of $a(\theta)$ using only the unrestricted model.

The first two procedures involve the estimation of the restricted model, implying the simultaneous solution of the economic model and the law of motion (given by the VAR representation), subject to eq. (7). Cho and Moreno (2004, 2006) propose a simultaneous estimation procedure of the above model using a maximum likelihood estimator. However, the non-linearity restrictions on the structural parameters could potentially involve multiple stationary or complex valued solutions, or even no solutions at all (i.e., no convergence of the iteration process). Accordingly, for testing purposes in this case, there would be a strong preference for using the unrestricted model and tests based on Wald statistics.[2] The Appendix reviews the asymptotic properties

[2]Note, however, that Wald statistics are not free from disadvantages, as compared to the other alternatives. As noted by Godfrey and Veall (1985a,1985b) and Gregory and Veall (1987), among others, Wald statistics are not invariant to the functional form of the restrictions, i.e., $a(\theta)$.

of Wald tests for time-series models under the GMM framework.

In the exercises carried out in this paper, the implementation of the Wald test is done through the following steps. First, we run the reduced form VAR(1) models (3) or (4), that is, without imposing any restriction.

Then we consider the structural model M2-NKM eq. (1), which can be written as:

$$\left\{ \begin{array}{c} \Delta\pi_t = \alpha_{AS} + \delta(E_t\pi_{t+1} - \pi_{t-1}) + \lambda(y_t + y_{t-1}) + \epsilon_{AS_t} \\ \Delta y_t = \alpha_{IS} + \mu(E_t y_{t+1} - y_{t-1}) + \phi(E_t\pi_{t+1} - r_t) + \epsilon_{IS_t} \\ \Delta r_t = \alpha_{MP} + (\rho - 1)r_{t-1} + (1-\rho)\beta E_t\pi_{t+1} + (1-\rho)\gamma y_t + \epsilon_{MP_t} \end{array} \right. \quad (8)$$

The next stage is to replace $E_t X_{t+1}$ by $\hat{X}_{t+1}$ resulting from the VAR models estimated in the first step, using either the results of (3) or (4) according to whether one is considering the case of M-Ca(M2, M2) or that of M-Cb(M2, M1). It can be noted that it is not possible to obtain consistent least-squares estimators of the structural parameters as $\hat{X}_{t+1}$ contains X_t. This is solved by running instrumental variables (IV) estimators where the endogenous variables are instrumented by $\hat{X}_{t-1}$. Consistency is guaranteed by the assumptions on ϵ.

Given the parameters of the VAR reduced form, represented by the matrix $\hat{\Omega}$, and using the corresponding expected values in equations (8) the structural parameters in those equations can be estimated, giving the three matrices $(\hat{A}_{11}, \hat{B}_{11}, \hat{B}_{12})$.Now, the nine non-linear restrictions can be written as:

$$\hat{A}_{11}\hat{\Omega}^2 - \hat{B}_{11}\hat{\Omega} + \hat{B}_{12} = 0_{3\times 3}.$$

Now, those parameter restrictions are expressed in extensive form as a 9×1 vector function, $a(\theta)$. Define $\theta = (\delta, \lambda, \mu, \phi, \rho, \beta, \gamma, \omega')'$ as the $p = 7 + 9$ parameters and the equality $a(\theta) = 0_{9\times 1}$ given by

$$a_1(\theta) = (\omega_{\pi\pi}^2 + \omega_{\pi y}\omega_{y\pi} + \omega_{\pi r}\omega_{r\pi}) - \lambda(\omega_{\pi\pi}\omega_{y\pi} + \omega_{yy}\omega_{y\pi} + \omega_{yr}\omega_{r\pi}) - \delta\omega_{\pi\pi} + (1-\delta) = 0,$$

$$a_2(\theta) = (\omega_{\pi\pi}\omega_{\pi y} + \omega_{\pi y}\omega_{yy} + \omega_{\pi r}\omega_{ry}) - \lambda(\omega_{\pi y}\omega_{y\pi} + \omega_{yy}^2 + \omega_{yr}\omega_{ry}) - \delta\omega_{\pi y} + \lambda = 0,$$

$$a_3(\theta) = (\omega_{\pi\pi}\omega_{\pi r} + \omega_{\pi y}\omega_{yr} + \omega_{\pi r}\omega_{rr}) - \lambda(\omega_{\pi y}\omega_{\pi r} + \omega_{yy}\omega_{yr} + \omega_{yr}\omega_{rr}) - \delta\omega_{\pi r} = 0,$$

$$a_4(\theta) = (\omega_{y\pi}\omega_{\pi\pi} + \omega_{yy}\omega_{y\pi} + \omega_{yr}\omega_{r\pi}) + \phi(\omega_{r\pi}\omega_{\pi\pi} + \omega_{ry}\omega_{r\pi} + \omega_{rr}\omega_{r\pi}) - \phi\omega_{\pi\pi} - \mu\omega_{y\pi} = 0,$$

$$a_5(\theta) = (\omega_{y\pi}\omega_{\pi y} + \omega_{yy}^2 + \omega_{yr}\omega_{ry}) + \phi(\omega_{r\pi}\omega_{\pi y} + \omega_{ry}\omega_{yy} + \omega_{rr}\omega_{ry}) - \phi\omega_{\pi y} - \mu\omega_{yy} + (1-\mu) = 0,$$

$$a_6(\theta) = (\omega_{y\pi}\omega_{\pi r} + \omega_{yy}\omega_{yr} + \omega_{yr}\omega_{rr}) + \phi(\omega_{r\pi}\omega_{\pi r} + \omega_{ry}\omega_{yr} + \omega_{rr}^2) - \phi\omega_{\pi r} - \mu\omega_{yr} = 0,$$

$$a_7(\theta) = -(1-\rho)\gamma(\omega_{y\pi}\omega_{\pi\pi} + \omega_{yy}\omega_{y\pi} + \omega_{yr}\omega_{r\pi}) + (\omega_{r\pi}\omega_{\pi r} + \omega_{ry}\omega_{yr} + \omega_{rr}^2) - (1-\rho)\beta\omega_{\pi\pi} = 0,$$

$$a_8(\theta) = -(1-\rho)\gamma(\omega_{y\pi}\omega_{\pi y} + \omega_{yy}^2 + \omega_{yr}\omega_{ry}) + (\omega_{r\pi}\omega_{\pi y} + \omega_{ry}\omega_{yy} + \omega_{rr}\omega_{ry}) - (1-\rho)\beta\omega_{\pi y} = 0,$$

$$a_9(\theta) = -(1-\rho)\gamma(\omega_{y\pi}\omega_{\pi r} + \omega_{yy}\omega_{yr} + \omega_{yr}\omega_{rr}) + (\omega_{r\pi}\omega_{\pi r} + \omega_{ry}\omega_{yr} + \omega_{rr}^2) - (1-\rho)\beta\omega_{\pi r} + \rho = 0.$$

Note that $a(\eta, \omega) = vec(A_{11}(\eta)\Omega^2(\omega) - B_{11}(\eta)\Omega(\omega) + B_{12}(\eta))$, then $\partial_{\eta_j} a(\eta, \omega) = vec(\partial_{\eta_j} A_{11}(\eta)\Omega^2(\omega) - \partial_{\eta_j} B_{11}(\eta)\Omega(\omega) + \partial_{\eta_j} B_{12}(\eta))$ and $\partial_{\omega_k} a(\eta, \omega) = vec(2A_{11}(\eta)\Omega(\omega)\partial_{\omega_k}\Omega(\omega) - B_{11}(\eta)\partial_{\omega_k}\Omega(\omega))$. The variance- covariance matrix to be used to apply the Wald statistics can be calculated in the way shown in the Appendix.

Table 1: Wald test statistics

Expectations →	M2 (π_t, y_t, r_t)		M1 (π_t, y_t, r_t, m_t)	
Period ↓	Wald stat	p-value	Wald stat	p-value
1959-2015	0.583	1.000	0.566	1.000
1959-1979	1.786	0.994	1.497	0.997
1980-2000	2.778	0.972	1.230	0.999
1990-2010	2.304	0.986	1.898	0.993
2000-2015	0.174	1.000	16.55	0.056

4 Results of the Wald tests

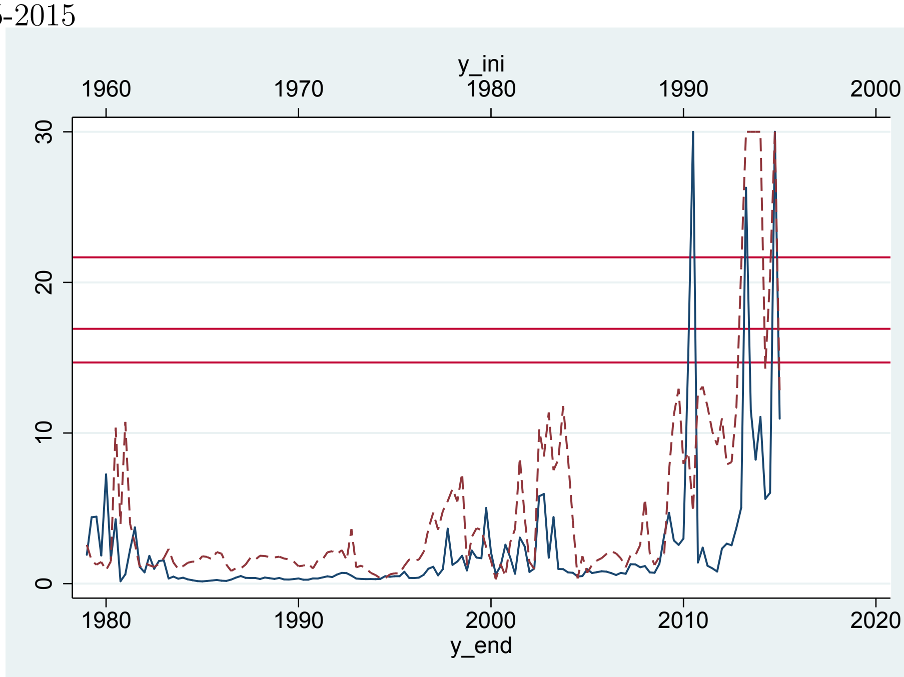
We estimate the model with U.S. quarterly data from 1959q1 to 2015q4. The inflation rate is the log first difference of the GDP deflator (source: Federal Reserve Bank of St. Louis). The Federal funds rate is the monetary policy instrument for model M2 (source: Board of Governors of the Federal Reserve System), while the money supply growth is measured by the log first difference of M2, seasonally adjusted (source: Board of Governors of the Federal Reserve System. The output gap is generated using the Hodrick-Prescott filter on the series for Gross National Product at 1996 constant prices (source: Federal Reserve Bank of St. Louis).

We first report Wald test results in Table 1 for the full sample, 1959-2015, then for several subsamples: 1959-1979, covering two decades before the Volcker era; 1980-2000, which happens to be the period on which Cho and Moreno (2004,2006), the M2 model, was originally estimated, and finally the latest period in the sample, 2000-2015, which includes observations during and after the recent latest financial crisis in the US. The sub-interval

1990-2010 is also considered, to see whether a change in behavior appears in this period. The tests are computed using either (π_t, t_y, r_t) or (π_t, y_t, r_t, m_t) to generate forecasts for the tests of M-Ca(M2-M2) and M-Cb(M2-M1), respectively.

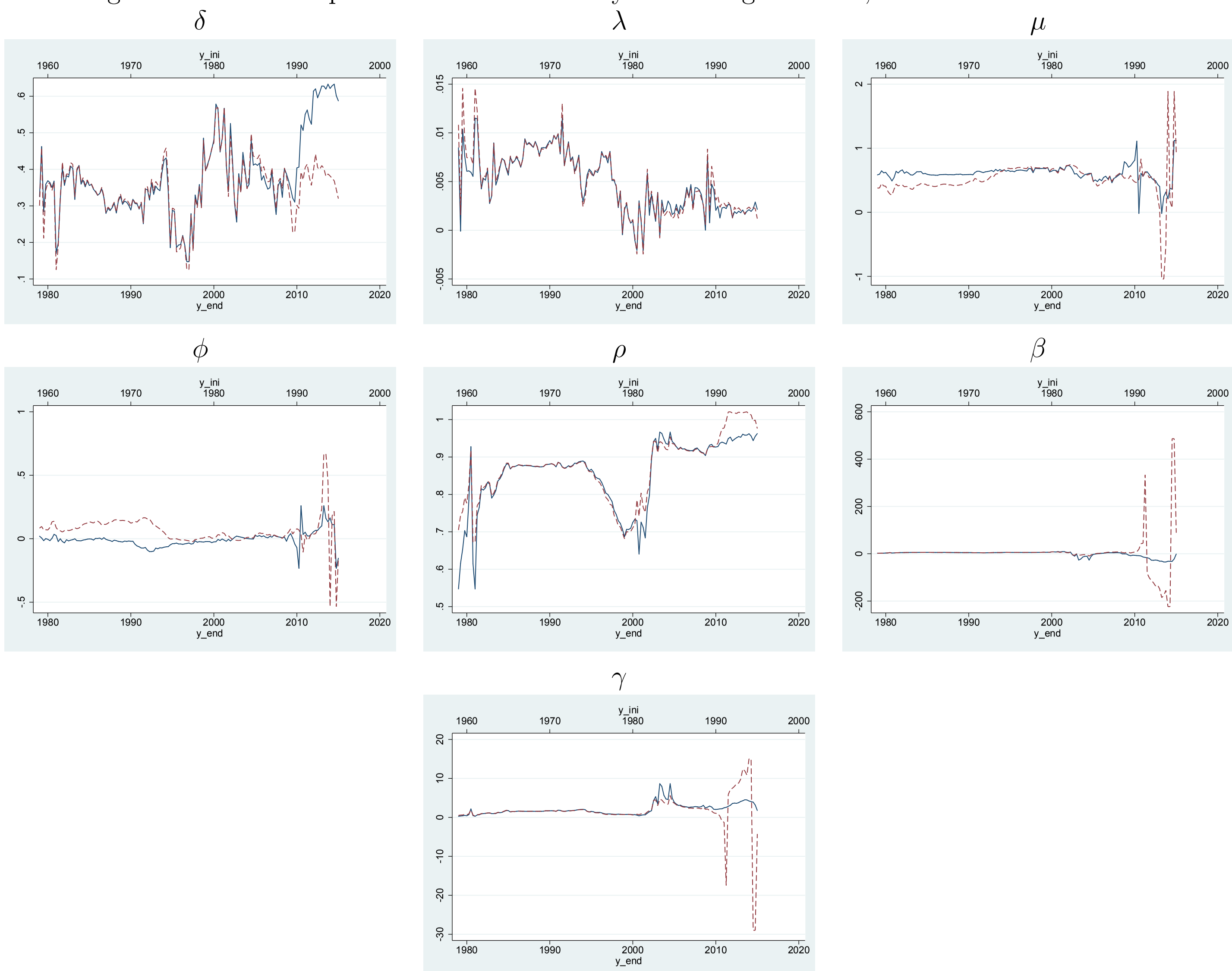
For the periods before the year 2000, the tests do not reject either of the model- consistent schemes: thus, they cannot make a sharp selection between alternative specifications of the expectations mechanisms, even though the policy regime changed substantially over the sample periods. The differentiation between the expectational alternatives appears more clearly for more recent periods after 2000, when there is a clear rejection of the anticipations based on the older- generation model, M-Cb(M2-M1), while M-Ca(M2-M2) cannot be rejected. With a closer look, using rolling-window estimates, the performance of the M2 model with M2- compatible expectations also becomes more problematic when observations pertaining to the financial crisis and its aftermath are included. Figure 1 reports the results of rolling-windows exercises with successive 20-year (80 quarters) samples, so that the estimates cover periods from 1959-1979 to 1995-2015. While no rejections are found before 2000, the Wald statistics rejects in many cases for samples that include post-2010 observations.

Figure 1: Wald test statistics for 20-years rolling-windows, 1959-1979 to 1995-2015



Notes: Solid line: forecasts using (π_t, y_t, r_t). Dashed line: forecasts using (π_t, y_t, r_t, m_t). Horizontal lines are the 1%, 5% and 10% critical values for a χ^2_9. Wald statistics are capped at a maximum value of 30. For the M-Ca(M2,M2) tests and the sub-samples 1990q3:2010q3 and 1994q4:2014q4 the Wald values are capped. For the M-Cb(M2,M1) tests the sub-samples ending in 2013:q2, 2013:q3, 2013:q4, 2014:q1 and 2014:q4 are capped.

Figure 2: Structural parameter estimates 20-years rolling-windows, 1959-1979 to 1995-2015



Notes: Solid line: forecasts using (π_t, y_t, r_t). Dashed line: forecasts using (π_t, y_t, r_t, m_t). In all figures the parameter values below the bottom 1% and above the upper 99% percentiles are capped.

Complementary results can be observed in Figure 2, which reports the estimated parameter values for each sub-sample. While the measured parameters did not show much inter- sample volatility in earlier periods, with values in the range of those found by Cho and Moreno (2006), after 2010 the estimated coefficients present a considerable sensitivity to the specific sample being considered, especially for the parameters of the aggregate demand function (i.e., (μ, ϕ)).

5 Concluding remarks

Since economic analysis changes over time, model- consistency of expectations is intrinsically a dated concept. Standard practice, which postulates that the expectations of agents are formed for all times (past, present and future) on the basis of the model currently being proposed by the economist, imputes to past decision- makers knowledge and views that the analyst did not hold at the time. If the implicit assumption is that the professional representations of the economy are somehow reflected in actual economic behavior, it would follow as a matter of logic that the schemes used by agents to anticipate expectations have been varying in correspondence with the evolution of influential theories and models. The notion may turn out to be more or less relevant, but in any case it has the feature of allowing to consider a particular learning dynamics, mirroring that of the economists in view (contemporaneous, in this case, rather than long defunct as in Keynes, 1936), and thus to depict expectational mistakes analogous to those of once- incumbent models. In this paper we have developed an exploratory exercise applying a model-consistency formulation which allows for the drift of expectations-forming tools as theories are modified. The preliminary results obtained suggest that the periods around macroeconomic crises can be particularly interesting periods to focus future research.

Appendix

Consider a set of m population moment conditions that will be used to construct GMM estimators (see, e.g., Hansen (1982) and Newey and West (1987)),

$$E\left[g(z,\theta_0)\right] = 0,$$

where $g(z,\theta)$ is an $m \times 1$ vector of functions of data and parameters, z is a $k \times 1$ random vector and θ is a $p \times 1$ vector of parameters.

When equation (**??**) is correct, the sample moments, i.e., $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g(z_t,\theta)$, should be close to zero when evaluated at $\theta = \theta_0$.

Let $W_T(\theta)$ be an $m \times m$ positive semi-definite matrix. Define the loss function $Q_T(\theta) = -\frac{1}{2} g_T(\theta)^\top W_T^{-1}(\theta) g_T(\theta)$. For asymptotic efficiency and to simplify the analysis we will assume $lim_{T\to\infty} W_T(\theta) = lim_{T\to\infty} Var[\sqrt{T} g_T(\theta)]^{-1} = W(\theta)$ and $W = W(\theta_0)$ (see Hansen (1982) and Newey and McFadden (1994)).

Let $\nabla_\theta g(z,\theta) = \partial g(z,\theta)/\partial\theta^\top$ be the $m \times p$ Jacobian matrix of $g(z,\theta)$, $G(\theta) = E[\nabla_\theta g(z,\theta)]$ and $G_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} \nabla_\theta g(z_t,\theta)$. Define the counterpart of the score (pseudo-score) as $q_T(\theta) = -G_T(\theta) W_T^{-1}(\theta) g_T(\theta)$, and $q_{j,T}(\theta)$ the $p_j \times 1$ sub-vector. Also, let $\Xi(\theta) = G(\theta)^\top W^{-1}(\theta) G(\theta)$, $\Xi_T(\theta) = G_T(\theta)^\top W_T^{-1}(\theta) G_T(\theta)$ and $\Xi = \Xi(\theta_0)$.

Consider a set of r restrictions given by the vector function $a(\theta)$. Define $\Lambda(\theta) = \nabla_\theta a(\theta)$ and $\Lambda = \Lambda(\theta_0)$, an $r \times p$ matrix of rank r.

We are interested in the null hypothesis $H_0 : a(\theta_0) = 0$ against $H_A : a(\theta_0) = d/\sqrt{T}$.

Consider the following assumptions in Newey and West (1987):

Assumption 1. *(i) The data $\{z_t\}_{t=1}^T$ are random vectors that are the first T elements of a strictly stationary stochastic process $\{z_t\}_{t=1}^{\infty}$ and has a measurable joint density function $f(z_1, ..., z_T, \theta)$ with respect to a measure $\Pi_{t=1}^T \nu$, where ν is a σ-finite measure on $\mathbb{R}^k$.*
(ii) For each $\theta \in \Theta \subset \mathbb{R}^p$, the elements of $g(z, \theta)$ are measurable in z and $\int g(z, \theta) f(z, \theta) d\nu = 0$.
(iii) The vector $g(z, \theta)$ is continuously differentiable on Θ, almost everywhere ν, and $a(\theta)$ is continuously differentiable on Θ. For each positive integer $n \geq 2$ the joint density $f(z_1, z_n, \theta)$ is continuous in θ almost everywhere $\nu \times \nu$. Also $\theta_0 \in int(\Theta)$ where Θ is compact.
(iv) There exist measurable functions $h_1(z)$ and $h_2(z)$, and $c > 1$, such that almost everywhere ν, and for all $\theta \in \Theta$ and $n \geq 2$,

$$|g(z, \theta)|^4 \leq h_1(z), \quad |\partial g(z, \theta)/\partial\theta|^2 \leq h_1(z),$$

$$f(z, \theta) \leq h_2(z), \quad f(z_1, z_n, \theta) \leq h_2(z_1) h_2(z_n),$$

$$\int [\gamma_1(z)]^c h_2(z) d\nu < +\infty, \quad \int h_2(z) d\nu < +\infty.$$

(v) There exist constants C, $\epsilon > 0$ such that either, (a) for all $\theta \in \Theta$, $\{z_t\}_{t=1}^{\infty}$ is uniform mixing with $\phi(n) \leq Cn^{-\epsilon}$, $\epsilon \geq max\{2, c/(c-1)\}$, (b) for all $\theta \in \Theta$, $\{z_t\}_{t=1}^{\infty}$ is strong mixing with $\alpha(n) \leq Cn^{-\epsilon}$, $\epsilon \geq max\{2, c/(c-1)\}$.

(vi) For all $\theta \in \Theta$, $E[g(z,\theta)] = 0$ only if $\theta = \theta_0$. Also G has rank p, the asymptotic covariance matrix of $\sqrt{T} g_T(\theta_0)$ is nonsingular, and Λ has rank r.

Define the unconstrained GMM estimator as

$$\hat{\theta}_T = \underset{\theta \in \Theta}{\text{argmax}}\, Q_T(\theta).$$

Assumption 1 and the results in Newey and West (1987) guarantees that $\hat{\theta}_T$ is consistent and asymptotically normal.

Note that by using the unconstrained estimator, a joint test for H_0 can be constructed as a Wald test as a simple application of the delta method. Following Newey and McFadden (1994, p.2220) and the application to time-series data in Newey and West (1987), under H_A , $\sqrt{T} a(\hat{\theta}_T) = d + \Lambda \Xi^{-1} G W^{-1/2} \mathcal{N} + o_p(1)$ where $\mathcal{N} \sim \mathcal{N}(0_m, I_m)$, and $\sqrt{T} a(\hat{\theta}_T) \xrightarrow{d} \mathcal{N}(d, \Lambda \Xi^{-1} \Lambda^{\top})$ as $T \to \infty$. Then

$$Wald_a(\hat{\theta}_T) = T a(\hat{\theta}_T)' (\Lambda \Xi^{-1} \Lambda^{\top})^{-1} a(\hat{\theta}_T) \xrightarrow{d} \chi_r^2(d' \Lambda \Xi^{-1} \Lambda d).$$

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